Algorithm for Determining the Potential Alpha Energy Concentration (PAEC) and the Equilibrium Equivalent Radon Concentration (EEC) of the Short-Lived Decay Products of Rn-222 and Rn-220 Using a Simple Measuring Device.

0. Introduction

The decay products of Rn-222 (hereinafter referred to as radon progeny) as well as those of Rn-220 (hereinafter referred to as thoron progeny) are heavy metal atoms (or ions) that occur in the air with varying size distributions. They are either attached to aerosol particles within a particle diameter[1] (d_p) range of 30 nm < d_p < 500 nm or exist as so-called clusters, which consist of the atom or ion surrounded by attached water molecules. These clusters are referred to as the free fraction of the progeny and have diameters of approximately 0.5 nm < d_p < 10 nm. Due to their small size, these clusters have significantly greater mobility and are thus more quickly deposited on solid surfaces in a room. Details regarding the behavior of these progeny in the air will not be further discussed here, and reference is made to the website *radon-folgeprodukte.de* and the literature (*Porstendörfer*[2]).

To assess the health risk associated with inhaling radon or thoron progeny, the measurement quantity known as the Potential Alpha Energy Concentration (PAEC) has been defined. The task in radiation protection measurements, therefore, is to detect this quantity by measurement. For the measurement, it is important to note that the progeny can be quantitatively deposited from an air stream onto a filter and that the alpha radiation emitted by the collected progeny can be measured with known efficiency. Various algorithms exist for calculating the PAEC, from the collected progeny, which depend on the measurement and collection cycles. For practical short-term measurements in radiation protection in mining, various methods have been developed, which, however, will not be further addressed here.

The algorithm described below is based on the measurement of radon and thoron progeny using the simple measuring device described in the construction guide on the website radon-folgeprodukte.de. However, any progeny measuring device that can implement the cycle described below can be used for the measurement. For measuring radon progeny without considering thoron progeny, no measurement cycle is required; rather, the continuous deposition of the progeny from an air stream onto a filter is sufficient. The following describes the measurement method along with the mathematical foundations for the separate determination of radon and thoron progeny. The determination of radon progeny without correction for thoron progeny is included.

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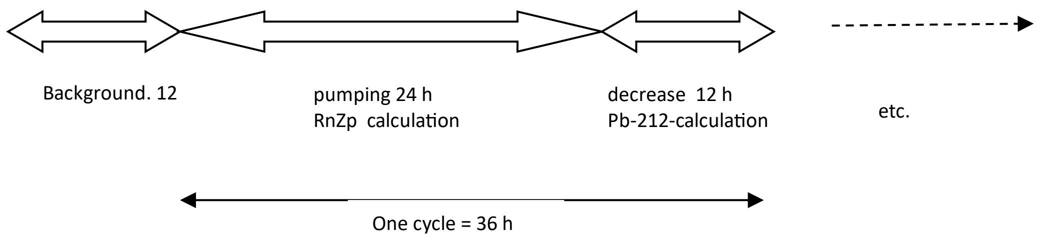
[1] What is meant here is the thermodynamic diameter. The difference between aerodynamic and thermodynamic diameter will not be discussed further.

[2] J. Porstendörfer *Properties and Behaviour of Radon and Thoron an Their Decay Products in the Air* J. Aerosol Sci Vol. 25,No. 2, S. 219-263, 1994

1. Description of the Measurement Cycle

The proposed measurement cycle consists of two parts. In the first part, the air to be measured is continuously drawn through a filter for 24 hours using a pump. In the second part, the pump is switched off for 12 hours. This decay phase is used to determine the thoron progeny, as the radon progeny has almost completely decayed after 3 hours, whereas the thoron progeny, due to the longer half-life of Pb-212 of approximately 10.6 hours, can be determined separately and without interference. The pulses caused by the emitted alpha particles are recorded integratively over 1 hour intervals, regardless of the pump's operation. The pulses registered during the 24-hour pumping cycle are used for the time-resolved determination of the PAEC of radon progeny. However, these must be subsequently corrected using the pulses caused by the thoron progeny. The development of the algorithm for thoron progeny assumes that their concentration remains constant during the pumping cycle. For the determination of average exposure values, this is insignificant, especially since the PAEC of thoron progeny is subject to significantly fewer variations than that of radon progeny. It is advisable to measure background after a filter change and before starting the measurements. For this purpose, a 12-hour interval is introduced before the actual measurement begins.

Time Schedule



In the following section 2, the algorithm for calculating PAEC and EEC for the short-lived radon progeny, without correcting for the contribution of thoron progeny, i.e., assuming that no thoron progeny is present in the air, is derived. In section 3, the algorithm for determining thoron decay products from the pulse measurement results during the 12-hour pump-off decay period is derived, along with the correction of the measured pulses during pumping to determine the PAEC of the radon progeny.

2. Calculation of PAEC and EEC of Short-Lived Radon Progeny Without Correction for the Contribution of Thoron Progeny

Symbols Used:

C_p: Potential Alpha Energy Concentration of Radon progeny,
C_{eec}: Equilibrium Equivalent Radon Concentration,
e_i : energy of alpha particles from nuclide ***i***,
n_i : Number of alpha particles detected from nuclide ***i***,
N_i: number of atoms of nuclide ***i***,
I_i: count rate,
Q: air stream,
C_i: Concentration of nuclide i in the air in atoms per unit volume
C_i^*: Activity concentration of nuclide i in the air $C_i^* = C_i \cdot \lambda_i$
λ_i: decay constant of nuclide ***i***,
A_i: activity of nuclide i on filter $A_i = N_i \cdot \lambda_i$,
V : Volume;
η_D: Detection efficiency of the emitted alpha particles
η_F: Efficiency of the deposition of radon progeny on the filter
η : Overall detection efficiency of the alpha particles $\eta = \eta_D \cdot \eta_F$
i : Index denoting the nuclides;
$i = 1\ (or\ a)$: Po-218
$i = 2\ (or\ b)$: Pb-214
$i = 3\ (or\ c)$: Bi/Po-214[3]
α : Index for the total alpha activity

2.1 General

By definition, PAEC is the energy that any mixture of short-lived radon progeny can emit in a unit volume of air until complete decay. All Po-218 atoms present in the volume element V, which is the concentration C_1, primarily emit alpha particles with energy e_1. Through the decay chain of Pb-214 and Bi-214, an additional alpha particle with the energy of Po-214, e_3, is emitted. All atoms of the nuclides Pb-214 and Bi-214 contribute secondarily to the PAEC only through Po-214.

$$C_p = e_1 \cdot C_1 + e_3 \cdot (C_1 + C_2 + C_3) \tag{2.1}$$

Based on this definition, one could determine C_p by collecting the nuclide mixture from a specific air volume V onto a filter, and then registering all decays from the start of pumping until complete decay after the pumping ends, multiplying by the energy of the alpha radiation. This represents the alpha energy that was potentially present in the air volume V. The alpha

[3] Po-214, due to its very short half-life, is always in radioactive equilibrium with Bi-214, and thus their activities can be considered equal.

particles are detected with an efficiency η by the detector and recorded as the ultimately counted pulses n_1 and n_3 . From this, the PAEC of the short-lived radon progeny is derived.

$$C_p = \frac{1}{V \cdot \eta} \cdot (e_1 \cdot n_1 + e_3 \cdot n_3) \tag{2.2}$$

For time-resolved routine measurements, this determination is not suitable. A method appropriate for routine measurements is the continuous air flow through a filter and measurement of the activity due to alpha radiation in equilibrium, which is typically reached approximately 3 hours after the start of filter exposure (pumping). This measurement method is employed by the described self-built device (see also details on the website *radon-folgeprodukte.de*).

2.2 Calculation of PAEC C_p

The mathematical relationships will be described in more detail here. The connection between filter activity and air concentration of each nuclide is calculated starting from the differential equation that describes the temporal behavior.

First, the activity on the filter during sampling needs to be calculated. For filter exposure under constant suction conditions (flow rate Q) and constant concentrations of radon decay products C_i, the filter activity of nuclide i is derived from the differential equation:

$$\frac{dN}{dt} = C_i \cdot Q + \lambda_{i-1} \cdot N_{i-1}(t) - \lambda_i \cdot N_i(t) \tag{2.3}$$

Equation (2.3) can then be solved step-by-step for the single nuclides, starting with Po-218 as the first member of the decay chain and following by the nuclides Pb-214 and Bi/Po-214. In the case of Po-218, the contribution N_{i-1} is set to 0. For the subsequent nuclides, the result from the previous nuclide is used in Equation (2.3). The result is the number of atoms on the filter N_i, and thus the activities $A_i = \lambda_i \cdot N_i$of the individual nuclides on the filter as a function of the pumping time t_p[4].

$$A_1 = Q \cdot C_1 \cdot \left(1 - e^{\lambda_1 \cdot t_p}\right) \tag{2.4}$$

$$A_2 = Q \cdot C_1 \cdot \left[\left(1 - e^{\lambda_2 \cdot t_p}\right) + \frac{\lambda_2}{\lambda_2 - \lambda_1} \cdot \left(e^{\lambda_2 \cdot t_p} - e^{\lambda_1 \cdot t_p}\right)\right] + Q \cdot C_2 \cdot \left(1 - e^{\lambda_2 \cdot t_p}\right) \tag{2.5}$$

$$A_3 = Q \cdot C_1 \cdot \left[\left(1 - e^{\lambda_3 \cdot t_p}\right) + \frac{\lambda_3}{\lambda_2 - \lambda_3} \cdot \left(e^{\lambda_2 \cdot t_p} - e^{\lambda_3 \cdot t_p}\right) + \frac{\lambda_2 \cdot \lambda_3}{\lambda_1 - \lambda_2} \cdot \left(\frac{e^{\lambda_1 \cdot t_p} - e^{\lambda_3 \cdot t_p}}{\lambda_3 - \lambda_1} - \frac{e^{\lambda_2 \cdot t_p} - e^{\lambda_3 \cdot t_p}}{\lambda_3 - \lambda_2}\right)\right] + Q \cdot C_2 \cdot \left[\left(1 - e^{\lambda_3 \cdot t_p}\right) + \frac{\lambda_3}{\lambda_2 - \lambda_3} \cdot \left(e^{\lambda_2 \cdot t_p} - e^{\lambda_3 \cdot t_p}\right)\right] + Q \cdot C_3 \cdot \left(1 - e^{\lambda_3 \cdot t_p}\right) \tag{2.6}$$

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[4] C. Feddersen, E. Dagen *Berechnung der Konzentration von Radon-Folgeprodukten in Luft* Report SAAS-257, 1980

For the total activity of alpha radiation during the exposure, without distinguishing between energies e_1 and e_3, the following applies:

$$A_\alpha(t) = A_1(t) + A_3(t) \tag{2.7}$$

After a sufficiently long time (t>3h), expressions (2.4) through (2.6) become:

$$A_1 = Q \cdot C_1 \tag{2.8}$$

$$A_2 = Q \cdot (C_1 + C_2)\,,$$

$$A_3 = Q \cdot (C_1 + C_2 + C_3) \tag{2.9}$$

And for the total alpha activity, the following applies:

$$A_\alpha = Q \cdot (2 \cdot C_1 + C_2 + C_3). \tag{2.10}$$

The equation (2.1) defining C_p changes without substantially additional error (< 5% relative) to the equation

$$C_p = e_m \cdot (2 \cdot C_1 + C_2 + C_3) \quad 2.11)$$

when using an average energy $e_m = 7{,}5\ MeV$ instead of e_1=6.09 MeV and instead of e_3=7.69. In deriving this average energy, it is taken into account that the number of Po-218 atoms is significantly smaller due to its shorter half-life compared to the subsequent nuclides.

Thus, from (2.10) and (2.11), the relationship between the desired measurement quantity, PAEC C_p, and the activity of alpha radiation on the filter A_α follows:

$$C_p = \frac{e_m \cdot A_\alpha}{Q} \tag{2.12}$$

The activity of alpha radiation on the filter $A_\alpha = \frac{I_\alpha}{\eta}$ is derived from count rate $I_\alpha = \frac{n_\alpha}{t_m}$.
Taking into account that the detection efficiency of a decay is $\eta = \eta_F \cdot \eta_D$, Equation (2.12) becomes:

$$\boldsymbol{C_p = \frac{e_m \cdot n_\alpha}{Q \cdot \eta_F \cdot \eta_D \cdot t_m}}\,. \tag{2.13}$$

Equation (2.13) forms the basis for calculating the potential alpha energy concentration from the measured pulse rate $I_\alpha = \frac{n_\alpha}{t_m}$.

(see also *radon-folgeprodukte.de*).

2.3 Calculation of EEC C_{eec}

The concept of equilibrium-equivalent radon concentration is advantageous for various analyses, particularly when assessing the degree of depletion of decay products from the air. This is important for model considerations regarding the behavior of decay products in relation to the so-called room model. The room model describes the concentration of decay products depending on various influencing parameters, such as air exchange in the room, deposition on surfaces, and attachment to aerosol particles. For further details, refer to the literature (*Porstendörfer, Jakobi*[5]) and the website *radon-folgeprodukte.de*.

By definition, the equilibrium-equivalent radon concentration (EEC) is the **hypothetical** radon concentration that, in equilibrium with all decay product nuclides, would have the same potential alpha energy concentration (PAEC) as the actual mixture of decay product nuclides present.

To derive the relationship between PAEC and EEC, the concentrations C_i of atoms per unit volume in equation (2.11) are replaced by the activity per unit volume. The relationship between atomic concentration and activity concentration is given by

$$C_i^* = \lambda_i \cdot C_i. \quad (2.14)$$

Thus, equation (2.11) can be expressed in the form

$$C_p = e_m \cdot \left(2 \cdot \frac{C_1^*}{\lambda_1} + \frac{C_2^*}{\lambda_2} + \frac{C_3^*}{\lambda_3}\right). \quad (2.15)$$

Since all activity concentrations of the decay product nuclides are, by definition, in equilibrium with the EEC, meaning they are also equal to each other, the following applies:

$$C_{eec} = C_1^* = C_2^* = C_3^* \quad (2.16)$$

and (eq. (2.15) changes to

$$C_p = e_m \cdot C_{eec} \cdot \left(2 \cdot \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3}\right) \quad (2.17)$$

or

$$C_{eec} = \frac{C_p}{e_m \cdot \left(2 \cdot \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3}\right)}. \quad (2.18)$$

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[5] J. Porstendörfer *Properties and Behaviour of Radon and Thoron an Their Decay Products in the Air*
J. Aerosol Sci Vol. 25,No. 2, S. 219-263, 1994
W. Jacobi *Activity and Potential a-Energy of 222Radon- and 220Radon-Daughters in different Air Atmospheres*
Health Physics, Vol. 22, S. 441-450, 1972

Taking into account eq. (2.12), eq. (2.18) changes to

$$C_{eec} = \frac{A_\alpha}{Q \cdot \left(2 \cdot \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3}\right)} . \qquad (2.19)$$

Using the primary measured quantities

- counts n_α,
- time of integration t_m and
- Efficiency of registration $\eta = \eta_D \cdot \eta_F$

Eq. (19) changes to

$$C_{eec} = \frac{n_\alpha}{Q \cdot \eta_D \cdot \eta_F \cdot t_m \cdot \left(2 \cdot \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3}\right)} . \qquad (2.20)$$

2.4 Conversion of PAEC to EEC

Starting from equation (2.1), the conversion factors between EEC and PAEC can be calculated exactly. The approximation used for the measurement-related relationship (the concept of average alpha energy) can be reversed. Equation (2.15) can then be expressed with the separated energies e_1 and e_3 in the form:

$$C_p = e_1 \cdot \frac{C_1^*}{\lambda_1} + e_3 \cdot \left(\frac{C_1^*}{\lambda_1} + \frac{C_2^*}{\lambda_2} + \frac{C_3^*}{\lambda_3}\right) . \qquad (2.21)$$

Since equation (2.16) applies to the EEC, the ratio of PAEC to EEC is:

$$\frac{\boldsymbol{C_p}}{\boldsymbol{C_{eec}}} = \boldsymbol{e_1} \cdot \frac{\mathbf{1}}{\boldsymbol{\lambda_1}} + \boldsymbol{e_3} \cdot \left(\frac{\mathbf{1}}{\boldsymbol{\lambda_1}} + \frac{\mathbf{1}}{\boldsymbol{\lambda_2}} + \frac{\mathbf{1}}{\boldsymbol{\lambda_3}}\right) \qquad (2.22)$$

On the right-hand side of equation (2.22), there are only natural constants, which establishes a fixed relationship between PAEC and EEC. Using the decay constants

$\lambda_1 = 0{,}0037877\ s^{-1}$

$\lambda_2 = 0{,}00043107\ s^{-1}$

$\lambda_3 = 0{,}00058053\ s^{-1}$

and the values of the energies

$e_1 = 6.0\ MeV$

$e_3 = 7.69\ MeV$

Equation (2.22) changes to

$$\frac{\boldsymbol{C_p}}{\boldsymbol{C_{eec}}} = \mathbf{1584.07}\ \boldsymbol{MeV \cdot s} + \mathbf{33116{,}1}\ \boldsymbol{MeV \cdot s}$$

(the reference to the volume ***V*** cancels out)

$\frac{\boldsymbol{C_p}}{\boldsymbol{C_{eec}}} = \mathbf{34700}\ \boldsymbol{MeV \cdot s}$ **. (**Dimension s is equivalent to $\frac{1}{Bq}$).

The common units for C_p are MeV/cm³ and nJ/m³. This gives the following conversion from EEC to PAEC:

$1\ Bq/m^3 = 0.0347\ MeV/cm^3$

$1\ Bq/m^3 = 5.559\ nJ/m^3$.

And for the conversion of $PAEC\ to\ EEC$:

$1\ MeV/cm^3 = 28.82\ Bq/m^3$

$1\ nJ/m^3 = 0{,}18\ Bq/m^3$.

2.5 Equation for the calculation of PAEC and EEC in common dimensions using continuous measurement

For equation (2.20), a numerical equation in common units can also be obtained. The expression in parentheses in equation (2.20) is:

4570,4 s.

Equation (2.20) in common form can be written as

$$C_{eec} = \frac{a \cdot n_\alpha}{Q \cdot \eta_F \cdot \eta_D \cdot t_m} \qquad (2.23)$$

where ***a*** is a constant that depends on the used units.

For the result in common units, the factor ***a*** is determined separately for each measurement times as follows:

- t_m $in\ min$ und
- t_m $in\ s$

	C_{eec}	C_p	
	Bq/m³	MeV/cm³	nJ/m³
t_m in min	0,219	0,00759	1,22
t_m in s	13,13	0,456	72,98

The factors for converting PAEC to EEC, which were derived in the previous section, were used. Equation (2.23) is the equation used in the "Evaluation" section of the website *radon-folgeprodukte.de*.

Equation (2.23) assumes, of course, that the detector registers only pulses emitted by decay products of Rn-222. This can be assumed as a first approximation for most measurement conditions in Germany. However, for more precise measurements, the always-present thoron decay products should be considered. This requires subsequent correction of the measured pulse rate. The basis for this correction is the knowledge of the average thoron decay product concentration during the pump cycle. Chapter 3 describes the calculation of the PAEC of thoron decay products from the pulse measurement results during the twelve-hour pump pause and, based on this, the correction of pulse rates for determining radon decay products.

3. Derivation of the algorithm for determining PAEC from Rn-220 decay products

The symbols used in this chapter differ slightly from those used in Chapter 2, and some additional symbols are required. Therefore, they are redefined here and used only for Chapter 3.

$N^i(t)$: Number of Pb-212atoms accumulated in the actual cycle at time ***t*** after start of pumping, present on the filter
N_E^i : Number of Pb-212atoms accumulated in the actual cycle at the end of pumping time
$A^i(t)$: Activity of Pb-212 on the filter accumulated in the actual cycle at time ***t*** after start of pumping, present on the filter
A_E^i : Activity Pb-212 accumulated in the actual cycle at the end of pumping time
$\dot{Q}$: air flow rate through the filter
C_{Pb}^i : Atomic concentration of Pb-212 in air during cycle ***i***
$\tilde{C}_{Pb}^i$: Activity concentration of Pb-212 in air during cycle ***i***
C_{eec}^{Tn} : Equilibrium Equivalent Thoron Concentration(Rn220)
C_{eec}^{Rn} : Equilibrium Equivalent Radon Concentration(Rn222)
C_p^{Tn} : Potential Alpha Energy Concentration of Thoron progeny (Rn220)
C_p^{Rn} : Potential Alpha Energy Concentration of Radon progeny (Rn222)
e_{Bi} : Energy of alpha particles emitted from nuclide Bi-212
e_{Po} : Energy of alpha particles emitted from nuclide Po-212
t_a : Time from the end of pumping to the start of counting
t_e : Time from the end of pumping to the end of pulse counting [6]
t_p : Pumping time (=24 h)
t_z : Cycle time (=36 h)
Z^i : Count sum between 3rd and 12th hour after Stop of pumping
$Z^j(i)$: Count sum of counting interval ***j*** in the cycle ***i***
n_{korr}^j : Count sum in counting interval ***j*** resulting from the thoron progeny in the current pumping cycle and from the residual activity of the thoron progeny from the previous cycle.
λ, λ_{Pb}: Decay constant of Pb-212= 0,0654 h^{-1}
λ_{Bi} : Decay constant of Bi-212= 0,686 h^{-1}
$\boldsymbol{i}$: Index of cycle number
$\boldsymbol{j}$: Index of counting interval

[6] This also marks the end of the cycle and the beginning of pumping for the next cycle

3.1 General

In the used measurement cycle, the determination the PAEC for thoron progeny is only possible as an average over the 24-hour pumping period. However, this poses no problem when assessing the contribution of thoron progeny to the total PAEC. It must first be stated that the PAEC of the thoron progeny is almost exclusively caused by Pb-212, which has a half-life of approximately 10.6 hours, with only a small contribution from Bi-212. For determining the PAEC, it is not significant whether Pb-212 or Bi-212 atoms are deposited on the filter. The deposited Pb-212 decays to Bi-212 on the filter.
Therefore, for the measurement of the PAEC, it is assumed, for simplicity, that the entire alpha activity on the filter is caused by Pb-212 in the air. Thus the task is to determine the activity concentration of Pb-212 in the air from the accumulated (alpha) activity. It must also be considered that the measured counts are not directly emitted by Pb-212, but result from the decay chain via Po-212 (approximately 65% with an energy of 8.78 MeV) and Bi-212 (about 25% with 6.05 MeV and 10% with 6.09 MeV). Due to their decay constants, these nuclides are in equilibrium with Pb-212 in the measurement cycle used, and the sum of their contributions corresponds to the Pb-212 activity. Since no spectrometry is performed in this measurement method, the distribution of different energies is irrelevant.
First, in section 3.2, the Pb-212 activity A_{Pb} on the filter is calculated as a function of the Pb-212 concentration (atoms) C_{Pb}; from this, in section 3.3, the sum of counts during the decay phases is determined. These sums establish the relationship between the counts totals and the Pb-212 concentration C_{Pb} and activity concentration $\tilde{C}^i_{Pb}$. It should also be noted that the calculated counts totals in the current cycle (***i***) must corrected by the residual activity collected in the previous cycles (***i-1***) and (***i***-2).
Furthermore, the count totals determined in this way can be used to correct the measured values for determining the PAEC of radon progeny (section 3.5).

3.2 Calculation of Pb-212-activity on the filter

First, the number of atoms of the nuclide Pb-212, $N^i(t)$, accumulated on the filter as a result of the pumping process during the current cycle (***i***) is calculated as a function of time.

$$\frac{dN^i}{dt} = \dot{Q} \cdot C^i_{Pb} - \lambda \cdot N^i \qquad (3.1)$$

$$N^i(t) = \frac{1}{\lambda} \cdot \dot{Q} \cdot C^i_{Pb}\left(1 - e^{-\lambda \cdot t}\right) \text{ mit } N^i(0) = 0 \qquad (3.2)$$

At the end of each pumping cycle ***i***, the accumulated activity of Pb-212 on the filter, A^i_E, is therefore:

$$A^i_E = \dot{Q} \cdot C^i_{Pb}\left(1 - e^{-\lambda \cdot t_p}\right) \qquad (3.3)$$

C^i_{Pb} is the concentration of Pb-212 (atoms per unit volume) in the air during the cycle number ***i***, which needs to be determined. For the calculation, this concentration is assumed to remain constant during the measurement.

3.3 Calculation of total counts during the decay phase

To determine the Pb-212 activity concentration, the count sum resulting from the decay of Pb-212 atoms collected on the filter (via Bi-212 and Po-212) is measured. The total number of alpha decays used for the measurement is the integral from t_a=3h to t_e=12h (after the end of pumping) and consists of the activity A_E^i ,collected during pumping in cycle $\boldsymbol{i}$, and the decaying contributions from the previous two cycles (***i***-1) and (***i***-2). The sum of the alpha decays from the respective cycles is obtained through the undisturbed superposition of these three contributions.

The alpha decays are registered by the detector with a detection probability η. Thus, there the relationship between the registered number of impulses Z and the activity A is as follows:

$$Z^i = \eta \cdot \int_{t_a}^{t_e} A^i(t)\, dt \qquad (3.4)$$

For the decaying activity $A^i(t)$of Pb-212 after the end of pumping in cycle ii, the following applies:

$$A^i(t) = A_E^i \cdot e^{-\lambda t} \qquad (3.5)$$

The resulting contribution to the impulse sum Z^i is therefore:

$$Z^i = \eta \cdot A_E^i \cdot \frac{1}{\lambda}\left(e^{-\lambda \cdot t_a} - e^{-\lambda t_e}\right). \qquad (3.6)$$

Using $A_E^i = Q \cdot C_{Pb}^i \cdot \left(1 - e^{-\lambda \cdot t_p}\right)$ Z^i will change to

$$Z^i = Q \cdot C_{Pb}^i \cdot \left(1 - e^{-\lambda \cdot t_p}\right) \cdot \eta \cdot \frac{1}{\lambda} \cdot \left(e^{-\lambda \cdot t_a} - e^{-\lambda t_e}\right) \qquad (3.7)$$

Similarly, the contribution from the residual activity of the previous cycle $(i-1)$ applies:

$$Z^{i-1} = \eta \cdot \int_{t_z+t_a}^{t_z+t_e} A_R^{i-1}(t)\, dt \qquad (3.8)$$

$$Z^{i-1} = Q \cdot C_{Pb}^{i-1} \cdot \left(1 - e^{-\lambda \cdot t_p}\right) \cdot \eta \cdot \frac{1}{\lambda}\left(e^{-\lambda \cdot (t_z + t_a)} - e^{-\lambda \cdot (t_z + t_e)}\right) \qquad (3.9)$$

and from the second-to-last cycle (*i*-2):

$$Z^{i-2} = \eta \cdot \int_{2t_z+t_a}^{2t_z+t_e} A_R^{i-2}(t)\, dt \qquad (3.10)$$

$$Z^{i-2} = Q \cdot C_{Pb}^{i-2} \cdot \left(1 - e^{-\lambda \cdot t_p}\right) \cdot \eta \cdot \frac{1}{\lambda}\left(e^{-\lambda \cdot (2 \cdot t_z + t_a)} - e^{-\lambda \cdot (2 \cdot t_z + t_e)}\right) \qquad (3.11)$$

The count sum measured between t_a and t_e after the current cycle i is then:

$$Z = Z^i + Z^{i-1} + Z^{i-2} \quad (3.12)$$

Using equations (3.7), (3.9), and (3.11), the desired activity concentration C^i_{Pb} for cycle $\boldsymbol{i}$ can be calculated, taking into account the results of the preceding cycles $\boldsymbol{i-1}$ and $\boldsymbol{i-2}$.

With the following notations:

$$E_0 = \left(1 - e^{-\lambda \cdot t_p}\right), \quad (3.13a)$$
$$E_1 = \left(e^{-\lambda \cdot t_a} - e^{-\lambda t_e}\right), \quad (3.13b)$$
$$E_2 = \left(e^{-\lambda \cdot (t_z \cdot + t_a)} - e^{-\lambda \cdot (t_z \cdot + t_e)}\right), \quad (3.13c)$$
$$E_3 = \left(e^{-\lambda \cdot (2 \cdot t_z \cdot + t_a)} - e^{-\lambda \cdot (2 \cdot t_z \cdot + t_e)}\right) \text{ and}$$
$$a = Q \cdot E_0 \cdot \frac{\eta}{\lambda} \quad (3.13d)$$

results:

$$Z^i = a \cdot C^i_{Pb} \cdot E_1 + a \cdot C^{i-1}_{Pb} \cdot E_2 + a \cdot C^{i-2}_{Pb} \cdot E_3 \quad (3.14)$$

and finally:

$$C^i_{Pb} = \frac{1}{a \cdot E_1} \cdot \left(Z - a \cdot C^2_{Pb} \cdot E_2 - a \cdot C^3_{Pb} \cdot E_3\right). \quad (3.15)$$

Equation (3.15) gives the particle concentration per volume. The activity concentration of Pb-212, $\tilde{C}^i_{Pb}$, is then:

$$\tilde{C}^i_{Pb} = \lambda \cdot C^i_{Pb} \quad (3.16)$$

Taking into account expressions (3.13a-d), the activity concentration of Pb-212 in the air, $\boldsymbol{\tilde{C}^i_{Pb}}$, for the current cycle $\boldsymbol{i}$ can be calculated using equations (3.15) and (3.16). The residual activity from the previous cycles is also considered.

3.4 Conversion into PAEC Thoron progeny products $\boldsymbol{C^{Tn}_p}$

The common unit of measurement for the activity concentration $\tilde{C}^i_{Pb}$ is Bq/m³. To convert it into the unit typically used for PAEC ($\boldsymbol{C^{Tn}_p}$), nJ/m³, the activity concentration must be multiplied by the energy of the emitted alpha particles. It should be noted that, in addition to Pb-212, a certain amount of Bi-212 is present in the collected activity, which is simplistically assumed here to be Pb-212.

$$C^{Tn}_{eec} \approx \tilde{C}_{Pb}$$

Thus, the measured activity concentration, following the definition of the equilibrium equivalent radon concentration (EEC), is also interpreted as EEC for thoron. However, the Bi-212 fraction must be considered in the definition equation.

$$C_p^{Tn} = C_{eec}^{Tn} \cdot e_m \left(\frac{1}{\lambda_{Bi}} + \frac{1}{\lambda_{Pb}} \right) \tag{3.18}$$

The alpha energy e_m emitted by the volume element until complete decay is 35% from Bi-212 and 65% from Pb-212. These proportions result from the decay scheme of the thoron progeny.

$$e_m = 0.35 \cdot e_{Bi} + 0.65 \cdot e_{Po} \, . \tag{3.19}$$

Because
$e_{Bi} = 6{,}06 \; MeV$ und $e_{Po} = 8{,}78 \; MeV$

follows

$$e_m = 7{,}8 \; MeV = 1.25 \cdot 10^{-3} nJ$$

The expression in parentheses in Equation (3.18) is 60,299 s. Thus, Equation (3.18) becomes

$$C_p^{Tn} = C_{eec}^{Tn} \cdot 1.25 \cdot 10^{-3} \, nJ \; \cdot 60299 \; s \tag{3.20}$$

Thus for $C_{eec}^{Tn} = 1 \; Bq/m^3$ a value of $C_p^{Tn} = 75{,}4 \; nJ/m^3$ results.

3.5 Correction of count rates for determining the PAEC of Radon progeny during the 24h – pump cycle

For the correction of the counts registered during the pumping phase, the concentration of thoron progeny determined later in the decay phase, which is approximately the concentration of Pb-212 (C_{Pb}), is used. The contribution from the Pb-212 activity accumulated during the current pumping cycle i, $A_{Pb}^i(t)$ is determined from the activity concentration $\tilde{C}_{Pb}^i$. For this contribution, the initial activity is $A_{Pb}^i(0) = 0$. From this, $A_{Pb}^i(t)$ is derived as follows:

$$A_{Pb}^i(t) = Q \cdot \frac{\tilde{\tilde{C}}_{Pb}^i}{\lambda} \cdot \left(1 - e^{-\lambda t}\right) . \tag{3.21}$$

From this, the total count Z^j is calculated, which is derived from the accumulated activity $A_{Pb}^i(t)$ during the respective measurement time j of the 24-hour pumping phase (1-hour interval). The counting of the measurement times during the pumping phase begins with $j = 1$. Therefore, the integration for the first measurement time j must be performed from $t_a = 0$ to $t_e = t_m$. For the measurement times, the following generally applies:

$t_a = (j - 1) \cdot t_m$ and $t_e = (j) \cdot t_m$ where t_m is always $t_m = 1$ h. Thus, for the interval j of cycle i, the following applies: :

$$Z^j(i) = Q \cdot \frac{\tilde{C}_{Pb}^i}{\lambda} \cdot \eta \cdot \int_{t_a}^{t_e} \left(1 - e^{-\lambda t}\right) dt = Q \cdot \frac{\tilde{C}_{Pb}^i}{\lambda} \cdot \eta \cdot \left[t + \frac{1}{\lambda} \cdot e^{-\lambda t} \right]_{(j-1)t_m}^{j \cdot t_m} . \tag{3.22}$$

Thus the result is $Z^j(i)$:

$$Z^j(i) = Q \cdot \frac{\tilde{C}_{Pb}^i}{\lambda} \cdot \eta \cdot \left[t_m - \frac{1}{\lambda} \cdot \left(e^{-\lambda(j-1)t_m} - e^{-\lambda j t_m}\right)\right]. \tag{3.23}$$

Due to the half-life of Pb-212 being 10.6 hours, the residual activity from the previous cycle $(i-1)$ cannot be completely neglected and must be accounted for in the correction. Analogous to Equation (3.23), the number of counts that must be subtracted from the current count for correction is:

$$Z^j(i-1) = Q \cdot \frac{\tilde{C}_{Pb}^{i-1}}{\lambda} \cdot \eta \cdot \left[t_m - \frac{1}{\lambda} \cdot \left(e^{-\lambda(j-1)(t_m+t_z)} - e^{-\lambda j(t_m+t_z)}\right)\right]. \tag{3.24}$$

To correct the count for the calculation of PAEC and EEC of radon progeny, the count n_α, which serves as the basis for the calculations according to Equations (2.13) and (2.20), must be adjusted by subtracting the count[7]

$$Z^j = Z^j(i) + Z^j(i-1) \tag{3.25}$$

This results in

$$\boldsymbol{n_\alpha^{korr}(j) = n_\alpha(j) - Z^j}\ . \tag{3.26}$$

4. Summary

The article describes a measurement cycle for determining the PAEC of radon and thoron progeny. It includes the mathematical foundations for evaluating count measurements and the relationship between the measurement parameters PAEC and EEC.

For calculating the PAEC of radon progeny using continuous measurement (without pump pauses) without correcting the count rates for the proportion of thoron progeny, Equation (2.13) is used. If correction is applied during the measurement cycle, the corrected count n_α^{korr} must be used according to Equation (3.26), taking into account Equations (3.23), (3.24), and (3.25).

The calculation of PAEC and EEC for thoron progeny is based on the counts measured during the pump pauses, which are the times t=3ht=3h to 12h12h. The activity concentration of Pb-212 in the air $\tilde{C}_{Pb}$ is calculated using Equations (3.15) and (3.16), and finally, the PAEC of the thoron progeny C_p^{Tn} is determined according to Equation (3.20), considering Equation (3.17).

Volkmar Schmidt, Lichtenau 09/12/2024

[7] For simplicity, only a past cycle is considered here.